A model of revolt for measuring cohesion during an internal conflict

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Abstract

This paper proposes a model of revolt in a group of agents with resources and power. An agent, its resources, and its power can be regarded as a province in a country, the tax collected in it, and its budget, respectively. Donor provinces transfer tax to recipient provinces and thus their power is less than their resources. Under no-revolt conditions, it is assumed that the revolt of donors is suppressed by punishment from recipients. Based on an analysis of data for the USSR for 1990, the Republic of China for 1912, and the Empire of Japan for 1883, all of which experienced internal conflict in the respective years, the intensity of punishment is found to be a measure of the cohesion of a country. The application of the proposed measure to criminal organizations is also examined.

Introduction

Revolts, mutinies, and other types of conflict are common in recorded history and have occurred in recent times. The outbreak and success or failure of conflicts greatly affect individual lives and society as a whole. Understanding the dynamics that underlie conflicts is essential for improving society and gaining insight into human civilization. Some previous studies on conflicts have been descriptive and qualitative, focusing on the peculiarities of certain events to understand the general aspects of conflicts [1–4]. This approach is reasonable because human society is complex and quantitative data on the dynamics of society are scarce. Other studies have used a game theory framework to analyze conflicts [5–8]. Although this approach allows for a detailed mathematical analysis, measuring the expected payoff is almost impossible for historical events.

This paper proposes a quantitative model with only a few measurable quantities that can be applied to historical events. This simple model is applicable to a wide range of conflicts, but may be insufficient for very complex conflicts. The quantities used in this model are the resources (revenue) and power (budget size) of each agent, which can be a province, a warlord, or a local government. Power is equal to resources if the total amount of tax is equal to the budget. However, they are not necessarily equal. For example, for a province, the tax collected might need to be transferred to the central government or other provinces. Donor agents are incentivized to revolt and recipient agents disincentivize donors from revolting with punishment. The intensity of punishment, which is the only parameter in the model, determines the dynamics of the model.

The rest of this paper is organized as follows. First, the model is formalized, and its properties are analyzed. Second, for some historical revolts and breakaways, the intensity of punishment is estimated, and its implications are discussed. Finally, the limitations, possible extensions, and potential applications of the model are discussed.

Model and analysis

Let us assume that there are n agents and that agent i ($1 \leq i \leq n$) has resources $r_i \geq 0$ and power $p_i \geq 0$. For a given agent, its resources are equal to its power if there is no interaction among agents. If the agents interact, power (*e.g.*, taxes, tributes, or grants) is transferred among them. For the set of agents, the sum of all resources is equal to that of power:

$$\sum_{i=1}^{n} r_i = \sum_{i=1}^{n} p_i. \tag{1}$$

Donors (*i.e.*, agents that transfer their power to other agents) want to revolt to be free from a tax. By revolting, a set of agents, $\mathcal{A}$, try to restore their total power to be equal to their total resources (*i.e.*, $\sum_{i\in\mathcal{A}} p_i = \sum_{i\in\mathcal{A}} r_i$). However, the recipients (*i.e.*, agents that receive power from other agents) want to suppress the revolt. Specifically, the recipients or loyalists commit themselves to damaging the revolters with their power by $\alpha \sum_{i\in\bar{\mathcal{A}}} p_i$, where $\bar{\mathcal{A}}$ is the complement of $\mathcal{A}$ (*i.e.*, the set of loyalists) and α is the intensity of punishment. Hence, to prevent agents from revolting,

$$\sum_{i\in\mathcal{A}} p_i \geq \sum_{i\in\mathcal{A}} r_i - \alpha \sum_{i\in\bar{\mathcal{A}}} p_i, \tag{2}$$

must hold for all $\mathcal{A}$. The left-hand side is regarded as the total utility if the agents in $\mathcal{A}$ do not revolt and the right-hand side is regarded as the total utility if they do revolt. Here, we assume that the utility is equal to the power. In the following, we assume that $0 < \alpha < 1$. We assume that the loyalists commit themselves to damaging the revolters at any rate even if their own resources could also be damaged. We ignore the loyalists' choice of whose resources to damage and assume that the revolters can redistribute resources and power among themselves after the conflict. Whether the revolters can successfully commit themselves to the redistribution is not considered here. Thus, with the margin defined as

$$m_{\mathcal{A}} = \sum_{i\in\mathcal{A}} p_i - \sum_{i\in\mathcal{A}} r_i + \alpha \sum_{i\in\bar{\mathcal{A}}} p_i, \tag{3}$$

the condition $m_{\mathcal{A}} \geq 0$ for all $\mathcal{A}$ guarantees security for the loyalists.

The following investigates the possible allocation of power under Eq. (2). The results are summarized in Fig. 1, which shows the range of attainable p_1 versus r_1. We define $R = \sum_{i=2}^{n} r_i$, $P = \sum_{i=2}^{n} p_i$, $\pi_{\mathcal{A}} = P^{-1} \sum_{i\in\mathcal{A}\setminus\{1\}} p_i$, and $\rho_{\mathcal{A}} = R^{-1} \sum_{i\in\mathcal{A}\setminus\{1\}} r_i$.

First, let us consider the case in which agent 1 maximizes its power. For $r_1 = 0$, the maximum attainable power is $p_1 = 0$ (filled circle in Fig. 1) because for $\mathcal{A} = \{2, \ldots, n\}$, $m_{\mathcal{A}} = -p_1 + \alpha p_1 \geq 0$ holds only if $p_1 = 0$.

Increasing r_1 allows for a larger p_1, resulting in a monopoly of power (*i.e.*, $p_1 = r_1 + R$ and $p_2 = \cdots = p_n = 0$ for a sufficiently large r_1). This monopoly satisfies Eq. (2) if $r_1 \geq (\alpha^{-1} - 1)R$ because

$$m_{\mathcal{A}} = r_1 + R - \rho_{\mathcal{A}} R - r_1 \geq 0 \tag{4}$$

for $1 \in \mathcal{A}$ and

$$m_{\mathcal{A}} = -\rho_{\mathcal{A}} R + \alpha(R + r_1) \geq (1 - \rho_{\mathcal{A}})R \geq 0 \tag{5}$$

for $1 \notin \mathcal{A}$. The filled triangle in Fig. 1 shows $r_1 = (\alpha^{-1} - 1)R$ and $p_1 = r_1 + R$.

For $0 < r_1 < (\alpha^{-1} - 1)R$, the maximum attainable power is $p_1 = \frac{1}{1-\alpha} r_1$. To prove this, we assume that Eq. 2 holds for all $\mathcal{A}$ with $\mathbf{p} = (p_1,\ \ldots,\ p_n)$ and $\mathbf{r} = (r_1\ \ldots,\ r_n)$ and we increase r_1 to $r_1 + \Delta r_1$ and p_1 to $p_1 + \frac{1}{1-\alpha}\Delta r_1$ and decrease p_i $(2 \le i \le n)$ to $p_i - \frac{\alpha}{1-\alpha}\Delta r_1 P^{-1} p_i$. Then, if $1 \in \mathcal{A}$,

$$\begin{aligned} m_{\mathcal{A}} =& p_1 + \frac{1}{1-\alpha}\Delta r_1 + \pi_{\mathcal{A}} P - \pi_{\mathcal{A}} \frac{\alpha}{1-\alpha}\Delta r_1 \\ & - r_1 - \Delta r_1 - \rho_{\mathcal{A}} R \\ & + \alpha(1-\pi_{\mathcal{A}})P - (1-\pi_{\mathcal{A}})\frac{\alpha^2}{1-\alpha}\Delta r_1 \\ \ge& \frac{1}{1-\alpha}\Delta r_1 - \pi_{\mathcal{A}}\frac{\alpha}{1-\alpha}\Delta r_1 - \Delta r_1 \\ & - (1-\pi_{\mathcal{A}})\frac{\alpha^2}{1-\alpha}\Delta r_1 \\ \ge& 0. \end{aligned} \tag{6}$$

If $1 \notin \mathcal{A}$,

$$\begin{aligned} m_{\mathcal{A}} =& \pi_{\mathcal{A}} P - \pi_{\mathcal{A}}\frac{\alpha}{1-\alpha}\Delta r_1 - \rho_{\mathcal{A}} R \\ & + \alpha p_1 + \frac{\alpha}{1-\alpha}\Delta r_1 \\ & + \alpha(1-\pi_{\mathcal{A}})P - (1-\pi_{\mathcal{A}})\frac{\alpha^2}{1-\alpha}\Delta r_1 \\ \ge& -\pi_{\mathcal{A}}\frac{\alpha}{1-\alpha}\Delta r_1 + \frac{\alpha}{1-\alpha}\Delta r_1 - (1-\pi_{\mathcal{A}})\frac{\alpha^2}{1-\alpha}\Delta r_1 \\ \ge& 0. \end{aligned} \tag{7}$$

Combined with the solution $p_i = r_i$ $(1 \le i \le n)$, which always satisfies Eq. 2 for all $\mathcal{A}$, at $r_1 = 0$, this means that there exists a solution that satisfies $p_1 = \frac{1}{1-\alpha} r_1$ for $0 < r_1 < (\alpha^{-1} - 1)R$. What is left is to show that there is no solution with a larger p_1. If a solution satisfies Eq. (2) and $p_1 > \frac{1}{1-\alpha} r_1$, using Eq. (7) yields a monopoly of power with $r_1 < (\alpha^{-1} - 1)R$, but this is impossible. Hence, $p_1 = \frac{1}{1-\alpha} r_1$ is the optimal solution.

Second, we consider the minimum power of agent 1. Let us set $p_1 = r_1 - \Delta r_1$ and $p_i = (1 + \Delta r_1 R^{-1}) r_i$ $(2 \le i \le n)$, where

$$\Delta r_1 = \min\left\{ r_1,\ \frac{\alpha}{1-\alpha} R \right\}. \tag{8}$$

The filled square in Fig. 1 shows $r_1 = \frac{\alpha}{1-\alpha} R$ and $p_1 = 0$. For $1 \in \mathcal{A}$,

$$\begin{aligned} m_{\mathcal{A}} =& \rho_{\mathcal{A}} R + \rho_{\mathcal{A}}\Delta r_1 + r_1 - \Delta r_1 - \rho_{\mathcal{A}} R - r_1 \\ & + \alpha(1-\rho_{\mathcal{A}})R + \alpha(1-\rho_{\mathcal{A}})\Delta r_1 \\ =& \alpha(1-\rho_{\mathcal{A}})R - (1-\rho_{\mathcal{A}})(1-\alpha)\Delta r_1 \end{aligned} \tag{9}$$

is positive for $\Delta r_1 < \frac{\alpha}{1-\alpha} R$ and zero for $\Delta r_1 = \frac{\alpha}{1-\alpha} R$. In other words, $m_{\mathcal{A}}$ is zero for $\Delta r_1 = \frac{\alpha}{1-\alpha} R$ with any $\mathcal{A}$ that contains agent 1, meaning that this is the minimum attainable value. For $1 \notin \mathcal{A}$,

$$\begin{aligned} m_{\mathcal{A}} =& \rho_{\mathcal{A}} R + \rho_{\mathcal{A}}\Delta r_1 - \rho_{\mathcal{A}} R \\ & + \alpha(1-\rho_{\mathcal{A}})R + \alpha(1-\rho_{\mathcal{A}})\Delta r_1 + \alpha r_1 - \alpha \Delta r_1 \\ =& \rho_{\mathcal{A}}(1-\alpha)\Delta r_1 + \alpha(1-\rho_{\mathcal{A}})R + \alpha r_1 \ge 0. \end{aligned} \tag{10}$$

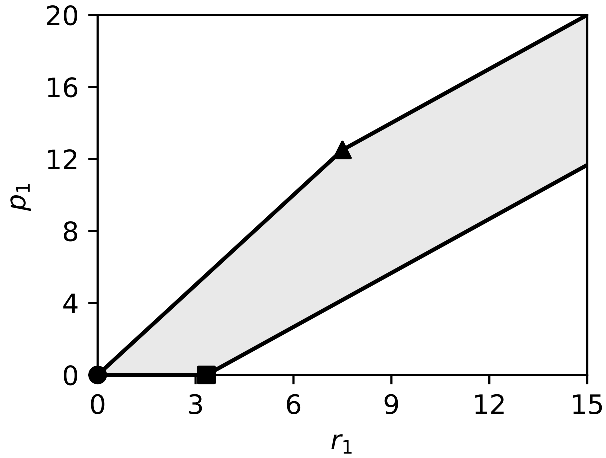

Fig 1. Range of p_1 for $\alpha = 0.4$ and $R = 5$. The feasible region is shown in light gray. The filled circle, triangle, and square indicate the coordinates $(0,\ 0)$, $((\alpha^{-1} - 1)R,\ \alpha^{-1}R)$, and $(\frac{\alpha}{1-\alpha}R,\ 0)$, respectively.

Finally, let us consider how to stabilize the country (*i.e.*, maximize $\underline{m} = \min_{\mathcal{A}} m_{\mathcal{A}}$). Under the assumption that the optimal solution $\underline{m}'$ is given by $\mathbf{p}' = (p_1',\ \ldots,\ p_n')$ for the maximization problem with $\mathbf{r}' = (r_1'\ \ldots,\ r_n')$, and that the optimal solution $\underline{m}''$ is given by $\mathbf{p}'' = (p_1'',\ \ldots,\ p_n'')$ for that with $\mathbf{r}'' = (r_1''\ \ldots,\ r_n'')$,

$$\begin{aligned}&\sum_{i\in\mathcal{A}}[tp_i' + (1-t)p_i''] - \sum_{i\in\mathcal{A}}[tr_i' + (1-t)r_i''] \\ &+ \alpha\sum_{i\in\underline{\mathcal{A}}}[tp_i' + (1-t)p_i''] \\ \geq\, & t\underline{m}' + (1-t)\underline{m}''\end{aligned} \tag{11}$$

is satisfied for $0 \leq t \leq 1$. This means that the optimal solution for $t\mathbf{r}' + (1-t)\mathbf{r}''$ is greater than $t\underline{m}' + (1-t)\underline{m}''$. Hence, $\underline{m}$ is a concave function of resources.

Application to historical events in USSR, China, and Japan

The previous section analyzed the model in which α and resources are constants and power is variable. This section estimates the value of α using the values of resources and power for historical events. Here, we assume that the agents are the provinces in a country, that the resources of a province are the amount of tax collected in the province, and that the power of a province is the budget of the province. Furthermore, we assume that a stable country satisfies Eq. (2) for all $\mathcal{A}$. Thus, the country destabilizes at the moment when $m_{\mathcal{A}}$ is zero for at least one $\mathcal{A}$. We can approximate the value of α in a year close to dissolution or civil war by the zero-crossing value of α.

To obtain the zero-crossing value of α, we need to check whether a set of resources, power, and α satisfies Eq. (2) for all $\mathcal{A}$. A brute-force check for all $\mathcal{A}$ requires $O(2^n)$ computational time. There is, however, a more computationally efficient method. If $l_i = (1-\alpha)p_i - r_i$ is negative for agent i, adding agent i to $\mathcal{A}$ makes $m_{\mathcal{A}}$ more negative. In contrast, if $l_i > 0$, eliminating agent i from $\mathcal{A}$ makes $m_{\mathcal{A}}$ more negative. Thus, for $\mathcal{A} = \{i \mid l_i < 0\}$, the margin $m_{\mathcal{A}}$ is the minimum for all $\mathcal{A}$ (*i.e.*, $\underline{m}$). If the margin is

Table 1. Values of α estimated for three historical events

Country (year)	α	Outcome
USSR (1990)	0.221	Dissolution
Republic of China (1912)	0.788	Prolonged civil war
Empire of Japan (1883)	0.950	Sustained unity

negative, Eq. (2) does not hold for at least one $\mathcal{A}$; if it is not negative, Eq. (2) holds for all $\mathcal{A}$. This method enables us to find the zero-crossing value of α using the bisection method because $m_{\mathcal{A}}$ and $\underline{m}$ are monotonically increasing functions of α. Because, by construction, $\underline{m}$ is a convex function of α, Newton's method can also be used.

There are two opposing interpretations of the estimated value of α. The first interpretation is that α is a measure of despotic oppression and thus a large α indicates that the country will dissolve. There might be a universal threshold of α, which when exceeded would indicate that civil war is likely. The other interpretation is that α is a measure of cohesion and unity and thus a large α indicates that the country will remain united. If this is the case, there is no universal threshold of α and thus α is characteristic to each country. To test these interpretations, we examine three examples, namely the USSR in 1990, the Republic of China in 1912, and the Empire of Japan in 1883.

The USSR dissolved in 1991, the last phase of the Revolutions of 1989 in the Eastern Bloc. In 1990, the governmental revenue of the union republics in the USSR consisted of turnover tax, personal income tax, enterprise tax, and grants from the Union government [9]. Part of the taxes collected in each union republic was retained by the union republic and the remainder was sent to the Union government. For a union republic, the tax collected in it was its resources and the sum of the tax retained in it and the grants from the Union government was its power. It was assumed that the difference between the total resources and the total power was sent to the Russian Soviet Federative Socialist Republic, the largest union republic. The value of α for this case was estimated to be 0.221. Because the tax on international trade, which went exclusively to the Union government, was ignored in the present calculation, this value is an upper bound of the estimate.

In 1912, the Qing dynasty collapsed, and the Republic of China was established. The Republic of China entered the Warlord Era in 1916. The provinces of the Republic of China sent two types of national tax, namely *jiekuan* and *zhuankuan*, to the central government [10]. It was assumed that the difference between the total resources and the total power was sent to Zhili province, which included Beijing. The value of α for this case was estimated to be 0.788.

Japan managed to suppress the largest rebellion in modern history, Satsuma Rebellion, in 1877. In 1883, the prefectures of Japan started publishing statistical tables in a unified format. The sum of the national tax, prefectural tax, and city tax collected in each prefecture was assumed to be the resources and the sum of the governmental subsidy, prefectural tax, and city tax was assumed to be power. Missing values were interpolated. It was assumed that the difference between the total resources and the total power was sent to Tokyo prefecture. The value of α for this case was estimated to be 0.950.

The α values estimated for these three cases are shown in Table 1. Dissolution occurred when α was a small value, it was avoided when α was a large value, and political turmoil without dissolution ensued when α was a moderate value. Thus, the estimated values are consistent with the second interpretation, namely that α is a measure of cohesion among people (allowing the government to set high taxes). In other words, α can be regarded as a measure of state capacity [11].

Discussion

This paper formulated and analyzed a model of revolt and estimated the value of α for three historical events. The estimated values suggest that α is a measure of the cohesion of agents.

Characterizing conflicts based on agents' resources and power and the intensity of punishment enables us to apply the model to other events. For example, the requisition of natural resources such as oil and gold could be understood using the proposed model. Furthermore, the model could be applied to the analysis of criminal organizations, religious sects, and political parties. Obtaining data on resources and power may be difficult for certain types of event, such as gang wars.

Nevertheless, the simplicity of the model allows us to obtain rough estimates for some cases. The National Police Agency in Japan stated in The White Paper on Police 1989 [12] that the total revenue of the yakuza (organized crime group in Japan) was 1.3019 trillion yen, that the total number of yakuza members was 86 552, and that the largest yakuza group, Yamaguchi-gumi, had 20 826 members in 737 subgroups. The White Paper wrote that the boss of a group collected 212.5 million yen per month from the subgroups. Under the assumptions that this group was Yamaguchi-gumi, that the share of revenue of Yamaguchi-gumi was identical to its share of members, that the money collected from the subgroups went to the subgroup of the boss, and that the resources of the subgroup of the boss are the average of the subgroups, we have $R = 1301.9 \times 20\,826/86\,552 \approx 313\,\text{billion yen}$, $r_1 = R/737 \approx 0.425\,\text{billion yen}$, and $p_1 = r_1 + 0.2125 \times 12 \approx 2.96\,\text{billion yen}$. Let us assume that this is the maximum attainable power, $p_1 = \frac{1}{1-\alpha} r_1$. This assumption is justified by the fact that 1989 was the year when a secession feud ended. The value of α for this case was estimated to be 0.857, which satisfies $r_1 < (\alpha^{-1} - 1)R$. This estimate of α lies between those of the Republic of China (1912) and the Empire of Japan (1883), suggesting rather high cohesion in yakuza groups. Consistent with this, it took more than a quarter century for the next major secession feud of Yamaguchi-gumi to break out.

There are some limitations of the proposed model and method. First, the time evolution of values is ignored. If the time series of the collected tax and budget of provinces are available, it will be interesting to examine whether the zero-crossing value of α increases or decreases toward the outbreak of a civil war. The time series of damage inflicted by revolts and loyalists should also be examined. Second, for simplicity, the model ignores the network structure of agents. Extending the model to consider agents interacting in a network would allow a wider range of phenomena to be analyzed. Geographical conditions and tribal disputes might exclude some types of coalition (*i.e.*, $m_{\mathcal{A}}$ might not be bounded for some $\mathcal{A}$). The heterogeneity of α should be taken into account. We might be able to investigate whether there could be a leader with less power than that of subordinates and whether tax rates converge to the same value for all subordinates. Third, although the collected tax and budget are regarded as the resources and power, respectively, in this paper, different quantities could be used. For example, the number of voters could be considered as resources for political parties. Then, the damage $-\alpha \sum_{i \in \mathcal{A}} p_i$ could be interpreted as reputation risk rather than damage caused by a direct attack. Additional quantities that specify agents could also be incorporated into the model. Fourth, a stricter formulation in the context of game theory is needed. The commitment to damaging the revolters and to redistributing resources ignored in the present formulation should be addressed.

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